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The evolution of correlation functions and power spectra in gravitational clustering

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ABSTRACT

Hamilton et al. proposed a simple formula relating the non-linear autocorrelation function of the mass distribution to the primordial spectrum of density fluctuations for gravitational clustering in an $\Omega=1$ universe. We present high-resolution N-body simulations which show this formula to work well for scale-free spectra $P(k)\propto k^n$ when $n\sim 0$, but not when $n<-1$. We show that a modified version of the formula can work well provided that its form is allowed to depend on n. Our modified formula is easy to apply and is a good fit to our N-body simulations which have $0\leq n\leq -2$. It can also be applied to non-power-law initial spectra such as that of the cold dark matter model by using the local spectral index at the current non-linear scale as the effective value of n at any given redshift. We give analytic expressions both for the non-linear correlation function and for the non-linear power spectrum.

Key words: galaxies: clusters: general – cosmology: theory – large-scale structure of Universe.

1 INTRODUCTION

According to the theory of gravitational instability, the clustering of matter in the Universe is determined by the power spectrum of primordial density fluctuations. Since present cosmic structures are highly non-linear on small scales, the relation between primordial and present spectra results from the interplay of complex processes. It is nevertheless highly desirable to find a simple formula which can relate the two at least approximately. Such a formula not only gives the two-point clustering properties of the evolved density field for any given initial model, but may also enable us to reconstruct the primordial density spectrum from the observed correlations of galaxies.

It is generally assumed that, in an Einstein–de Sitter universe, structure will grow self-similarly in time if the primordial spectrum has a power-law form:

$$P(k)=Ak^n, \tag{1}$$

where A is a constant and $4>n>-3$. For a given spectral index n, the correlation functions at different epochs are then related by a simple scaling relation (see Efstathiou et al. 1988, hereafter EFWD). Realistic power spectra are not pure power laws, but we will show that their evolution between the linear and highly non-linear regimes is similar to that of a power-law spectrum with an effective index $n_{\rm eff}$ where we define $n_{\rm eff}$ by

$$n_{\rm eff}=\left[\frac{{\rm d}\ \ln P(k_0)}{{\rm d}\ \ln k_0}\right]_{k_0=1/r_0}, \tag{2}$$

where r_0 is the radius of the top-hat window in which the rms mass fluctuation is unity.

Hamilton et al. (1991, HKLM) made an ansatz for the relation between the linear, and the evolved (or non-linear) average two-point correlation functions $\bar{\xi}_{\rm L}$ and $\bar{\xi}_{\rm E}$:

$$\bar{\xi}_{\rm E}(R)=F[\bar{\xi}_{\rm L}(R_0)], \qquad R_0=[1+\bar{\xi}_{\rm E}(R)]^{1/3}R. \tag{3a}$$

Here

$$\bar{\xi}(r)=(3/r^3)\int_0^r y^2\xi(y)\ {\rm d}y,$$

and F is a universal function assumed to be independent of the initial spectrum. This ansatz relates $\bar{\xi}_{\rm E}$ at a given length-scale to $\bar{\xi}_{\rm L}$ at a larger scale, thus taking account of the fact that non-linear density fluctuations shrink in comoving coordinates as evolution proceeds. A remarkable aspect of this ansatz is that the same function F appeared to work for all epochs and initial spectra (see also Nityananda & Padmanabhan 1994). HKLM showed that their ansatz is in good agreement with the N-body simulations of EFWD, and used these data to determine the functional form of F. Motivated by this work, Peacock & Dodds (1994, PD) studied a similar

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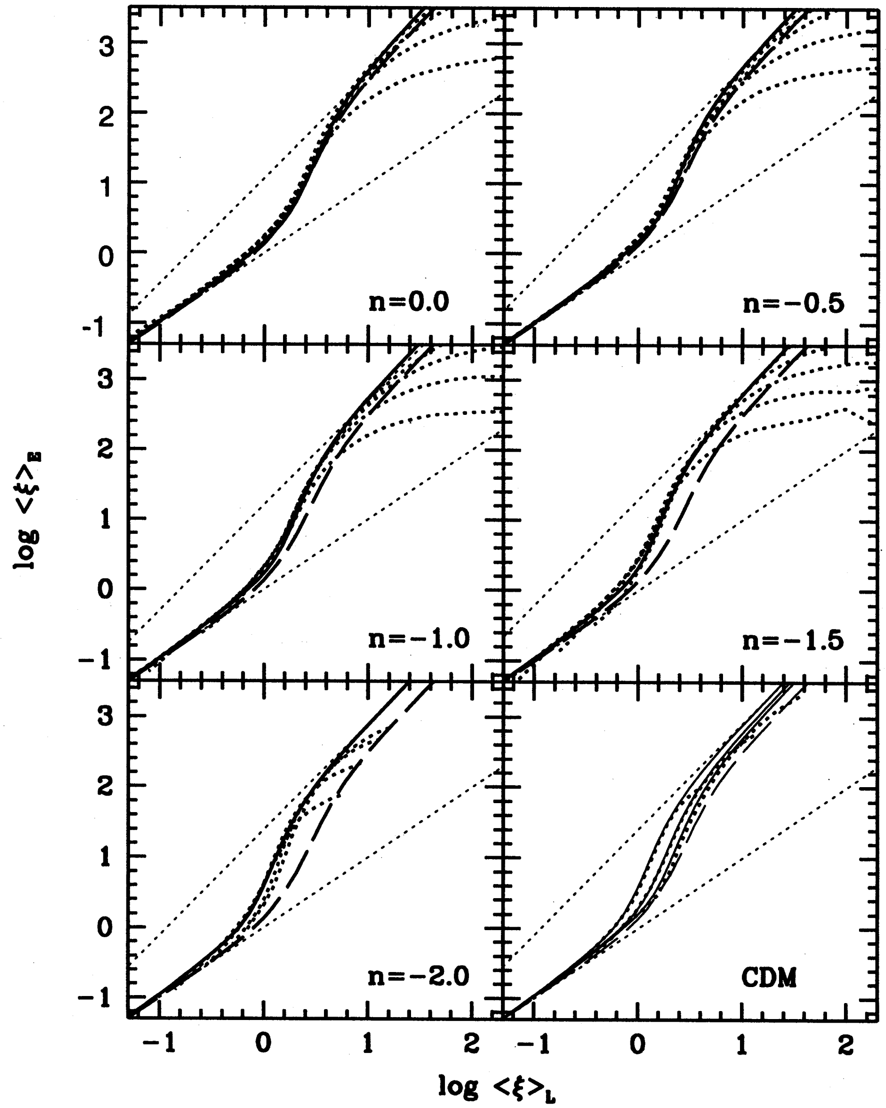

Figure 1. The evolved average mass correlation function $\bar{\xi}_E(R)$ as a function of the linear average mass correlation function $\bar{\xi}_L(R_0)$. Note that these two functions are calculated at two different scales, as discussed in the text. The dotted curves show the results derived from N-body simulations of different spectra. Results are shown for four different expansion factors a; a curve that flattens earlier corresponds to a lower value of a. The expansion factors are $a=(6.10, 14.91, 36.86, 90.88)$ for $n=0$, $(4.52, 9.53, 20.22, 42.82)$ for $n=-0.5$, $(3.34, 6.08, 11.08, 20.20)$ for $n=-1$, $(2.47, 3.86, 6.07, 9.52)$ for $n=-1.5$, $(1.14, 1.62, 2.29, 2.72)$ for $n=-2$, and $(0.2, 0.5, 1.0)$ for CDM. Long-dashed curves show the fitting formula of HKLM. Solid curves show the results of our improved formula (see text). The two light dotted lines show the expected behaviour of the relation in the linear and stable clustering (highly non-linear) regimes.

ansatz for the dimensionless power spectrum:

$$\Delta_E(k)=\Phi[\Delta_L(k_0)], \qquad k_0=[1+\Delta_E(k)]^{-1/3}k, \tag{3b}$$

where $\Delta_E(k)=4\pi k^3 P_E(k)$ (with P_E being the evolved power spectrum), $\Delta_L(k_0)=4\pi k_0^3 P(k_0)$, and Φ is a universal function. The relations in equations (3) are independent of the form of the initial spectrum P, but the authors noted that for $n=-2$ their formulae were not in good agreement with the simulation data. Such formulae are extremely useful, so it is important to check their validity for a wide range of initial spectra by using numerical simulations with better resolution than the old EFWD models.

This paper provides such a check. We find that equations (3a) and (3b) both work remarkably well for power-law spectra with $n\sim 0$, but that they are seriously in error for $n<-1$. This deficiency can be rectified by making the functional forms of F and Φ depend on n. We provide improved formulae which include a simple n-dependence and show how they can be applied to non-power-law models such as the cold dark matter (CDM) model. Section 2 presents results from several N-body simulations and compares them with the formulae of HKLM and PD. We present our improved model in Section 3 and give our conclusions in Section 4.

2 NON-LINEAR EVOLUTION AND THE UNIVERSAL SCALING ANSATZ

In this paper, we give simulation results for six different spectra. One is the standard CDM spectrum. The other five are power-law spectra with $n=0$, -0.5, -1, -1.5 and -2. All simulations assume an Einstein–de Sitter universe.

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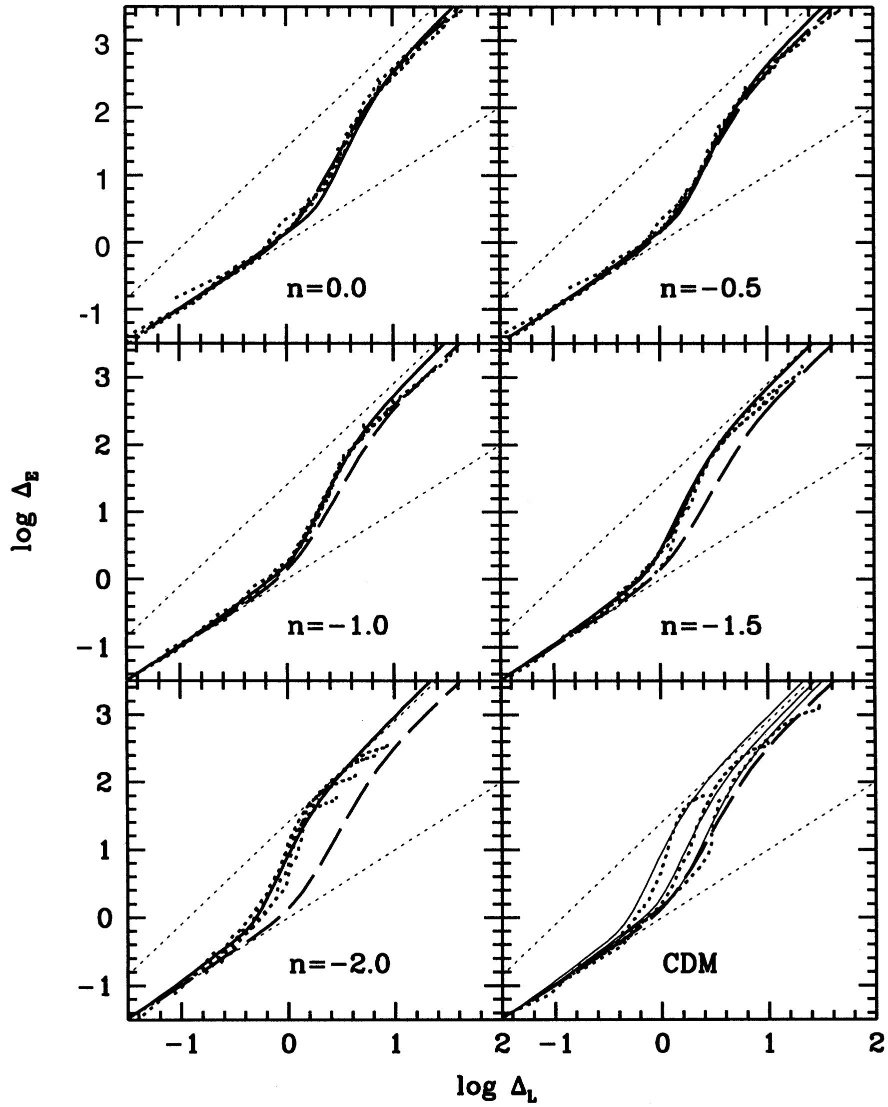

Figure 2. The evolved dimensionless power spectrum $\Delta_E(k)$ as a function of the linear dimensionless power spectrum $\Delta_L(k_0)$. Note that these two functions are calculated at two different scales, as discussed in the text. The dotted curves show the results derived from the same *N*-body simulations as described in Fig. 1. Long-dashed curves show the fitting formula of PD. Solid curves show the results of our improved formula (see text). The two light dotted lines show the expected behaviour of the relation in the linear and stable clustering (highly non-linear) regimes.

These simulations were performed using high-resolution particle–particle/particle–mesh (P^3M) codes. For models with $0 \geq n \geq -1.5$, the code is the same as in EFWD, but was run with more particles (100^3, compared with 32^3) and higher force resolution ($L/2500$, compared with $L/200$, where L is the side of the computational box). The power spectrum is normalized as described in EFWD.

The $n=-2$ run was performed by E. Bertschinger using an adaptive P^3M code. It followed 128^3 particles with a force resolution of $L/2560$. The CDM simulation, performed by Gelb & Bertschinger (1994), is also a P^3M simulation. It has 144^3 particles, $\Omega=1$, $H_0=50$ km s^{-1} Mpc^{-1}, $L=100$ Mpc and a force resolution of 65 kpc. It is normalized so that σ_8 (the linear rms mass fluctuation in a sphere of radius 16 Mpc) is unity when the expansion factor $a=1$. For the $n=-2$ and CDM simulations we have checked our results against other independent simulations – for $n=-2$ against an ensemble of three $N=256^3$ PM simulations, and for CDM against an $N=100^3$ P^3M simulation in a 50-Mpc box. In all cases the results agree to better than about 20 per cent in the critical transition between linear and non-linear regimes (see below).

The dotted curves in Fig. 1 show $\bar{\xi}_E(R)$ as a function of $\bar{\xi}_L(R_0)$. In each case we show the results for four different expansion factors as detailed in the figure caption. For each n, the curves for different a are close to each other, thus demonstrating that clustering is self-similar in the power-law models (see Jain & Bertschinger 1995 for a detailed examination of self-similar scaling for $-3<n<-1$). The flattening of the curves at high amplitudes, which is more severe for earlier output times, reflects the small-scale resolution limit. These results show that the $\log \bar{\xi}_E(R)$–$\log \bar{\xi}_L(R_0)$ relation depends on n and is a good fit to the HKLM formula (the long-dashed curve in each panel) only for $n \sim 0$. It misses the simulation results by factors of up to 3 and 10 around $\bar{\xi}_L \sim 1$ for $n=-1.5$ and -2 respectively. A similar failure is also seen in the CDM case where results for different a (or σ_8) differ substantially from each other. At $a=0.2$ the discrep-

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ancy around $\bar{\xi}_L \sim 1$ is almost a factor of 10. The effective power indices $n_{\rm eff}$ are -0.7, -1.3 and -2 for $a=1$, 0.5 and 0.2 respectively. The dependence on $n_{\rm eff}$ is strikingly similar to that for the corresponding power-law spectra.

The dotted curves in Fig. 2 show $\Delta_E(k)$ as a function of $\Delta_L(k_0)$ for the same simulations as in Fig. 1. The self-similarity of clustering is again verified for the power-law models, though the results for $n=-2$ have some scatter. Again the relation between $\Delta_E(k)$ and $\Delta_L(k_0)$ depends on n. The long-dashed curves in each panel show the fit proposed by PD. As for the HKLM formula, the PD formula works well only for $n \sim 0$. For $n<-1$ and for the CDM spectrum, there are substantial deviations from the simulation data. This is not surprising because, as pointed out by PD, their formula reproduces the HKLM formula very accurately for $\Omega=1$, and so reproduces its shortcomings.

3 AN IMPROVED MODEL

Since for given n the correlation functions and dimensionless power spectra obey scaling relations of the form (3), we can write both in the more general form $y=f_n(x)$. From Figs 1 and 2 we see that, although the relation between the evolved and unevolved quantities differs for different spectra, the shapes of the various curves are similar. This suggests that the n-dependence can be scaled away by a simple shift in the log–log plane. From linear theory and from the stable clustering hypothesis, respectively, we know that f_n has the following asymptotic behaviour: $f_n(x)=x$ as $x \to 0$ and $f_n(x) \propto x^{3/2}$ as $x \to \infty$. Thus the shift can be made only in the direction $y=x$. This suggests the ansatz

$$\frac{\bar{\xi}_E(R)}{B_\xi(n)} = F\left[\frac{\bar{\xi}_L(R_0)}{B_\xi(n)}\right] \tag{4a}$$

and

$$\frac{\Delta_E(k)}{B_\Delta(n)} = \Phi\left[\frac{\Delta_L(k_0)}{B_\Delta(n)}\right], \tag{4b}$$

where $B_\xi(n)$ and $B_\Delta(n)$ are constants depending on n, and F and Φ remain independent of n. Since F and Φ are non-linear functions, $B_\xi(n)$ and $B_\Delta(n)$ are generally not equal, although Δ and $\bar{\xi}$ are proportional to each other. The simulation data can be fitted reasonably well by the following simple empirical formula:

$$B_\xi(n) = \left(\frac{3+n}{3}\right)^{0.8}; \tag{5a}$$

$$B_\Delta(n) = \left(\frac{3+n}{3}\right)^{1.3}. \tag{5b}$$

As n approaches -3, both $B_\xi(n)$ and $B_\Delta(n)$ tend to zero and our approach breaks down. This reflects the breakdown of hierarchical clustering as $n \to -3$.

In the first panel of Fig. 1, the solid curve shows our best fit to the simulation results for $n=0$. Our result is

$$F(x) = \frac{x+0.45x^2-0.02x^5+0.05x^6}{1+0.02x^3+0.003x^{9/2}}. \tag{6a}$$

The inverse of this can be approximated to better than 4 per cent over the range of interest by

$$\hat{F}(y) = y\left(\frac{1+0.036y^{1.93}+0.0001y^3}{1+1.75y-0.0015y^{3.63}+0.028y^4}\right)^{1/3}. \tag{6b}$$

The solid curves in the other panels of Fig. 1 are obtained from equation (4a), with F given by (6a) and $B_\xi(n)$ given by (5a). For the CDM case, we have used the effective power indices given above. It is remarkable that this simple formula works accurately in all cases. For spectra with $n<-1$ and for the CDM spectrum, it makes a substantial improvement over the formula given by HKLM. For all six spectra, the N-body points (the mean of the four time outputs in the case of $n=-2$) agree with our fit to better than 20 per cent for $\bar{\xi}_E \sim 1$–100, provided that it is within the range of numerical validity of the simultion for the given time. Since the scatter between different simulations of a given spectrum is typically 15 per cent, this fit is sufficiently accurate.

The solid curve in the first panel of Fig. 2 is given by the fitting formula

$$\Phi(x) = x\left(\frac{1+0.6x+x^2-0.2x^3-1.5x^{7/2}+x^4}{1+0.0037x^3}\right)^{1/2}. \tag{7a}$$

The inverse of this function is fitted to an accuracy of better than 2 per cent over the range of interest by

$$\hat{\Phi}(y) = y\left(\frac{1+0.22y^{1.86}+0.000\,345y^3}{1+1.58y+0.05y^3+0.072y^4}\right)^{1/3}. \tag{7b}$$

The solid curves in other panels are obtained from equation (4b), with Φ given by (7a) and $B_\Delta(n)$ given by (5b). For the CDM spectrum, we have again used the effective power index $n_{\rm eff}$. We have transformed the $\Delta_E(k)$–$\Delta_L(k_0)$ relations represented by the solid curves to the corresponding $\bar{\xi}_E(R)$–$\bar{\xi}_L(R_0)$ relations. In all cases the transforms agree with the $\bar{\xi}_E(R)$–$\bar{\xi}_L(R_0)$ relations shown by the solid curves in Fig. 1.

Fig. 2 shows that our model also works for the dimensionless power spectrum. There are small differences in the shapes of the N-body curves for different spectra – as n becomes more negative, the $\Delta_E(k)$–$\Delta_L(k_0)$ curve rises more sharply in the regime where $\Delta_L(k_0) \sim 1$. For CDM the effect is pronounced at high redshifts. This suggests that one can do marginally better by introducing an n-dependence in the shape of the curve as well. There is also a hint that for CDM-like spectra the curve steepens more than the corresponding power-law case. However, our fit is adequate given the scatter in the simulation data, as the discrepancy between the fit and the simulations rarely exceeds the level of the scatter in the simulation data, which is typically 15–20 per cent. This agreement can be compared with the discrepancies of the PD fit, which is almost a factor of 10 below the mean of the simulation data for $n=-2$ and for CDM at $a=0.02$ around $\Delta_E(k) \sim 10$.

We note that the lowest order non-linear term in the fitting formulae (6a) and (7a) is of order x^2. However, this term is significant only in the range $1<\bar{\xi}_L$, $\Delta_L<4$ – at smaller amplitudes linear theory is sufficiently accurate, and at larger amplitudes higher order terms dominate the fitting formulae. Whether second-order perturbation theory agrees

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with our formulae within the range of dominance of the x^2 term can be ascertained by comparing the coefficient of this term with the predictions of perturbation theory. The outcome is dependent on the spectrum, because perturbation theory does not treat different spectra in the same way as the scaling ansatz. For non-power-law spectra the outcome also depends on the time at which the comparison is made: for example, for the CDM spectrum the agreement is very good at high redshift, but for $z \lesssim 1$ perturbation theory significantly overestimates the non-linear spectrum. Since our formulae fit the simulation results, these features can be inferred from previous work in which the second-order power spectrum is compared with *N*-body simulations (Jain & Bertschinger 1994, and references therein).

4 DISCUSSION

For a given linear power spectrum $P(k_0)$, the effective power index $n_{\rm eff}$ can be obtained from equation (2). The linear dimensionless power spectrum $\Delta_{\rm L}(k_0)$ and the linear average correlation function $\bar{\xi}_{\rm L}(R_0)$ can be calculated directly from $P(k_0)$. The evolved dimensionless spectrum $\Delta_{\rm E}(k)$ and the evolved average correlation function $\bar{\xi}_{\rm E}(R)$ can then be calculated from equations (4b) and (4a), respectively, with F given by equation (6a), Φ given by equation (7a), and $B(n)$ given by equations (5). To obtain the linear spectrum from the evolved spectrum (or correlation function), one can use equations (6b) and (7b). In this case, an iterative procedure may have to be used, because the effective power index is unknown a priori. As noted by PD, the effects of redshift-space distortions and biasing also need to be considered in such reconstructions. For other applications, such as using the non-linear power spectrum to compute the statistics of photon trajectories, our results can be used directly (Seljak 1995). It would clearly be useful to extend these results to cosmological models with $\Omega < 1$ or those that involve a hot dark matter component (see PD).

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