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COBE background radiation anisotropies and large-scale structure in the Universe

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SUMMARY

The microwave background anisotropies detected by the *COBE* DMR experiment provide our first detailed information about primordial fluctuations. Their properties suggest that the Universe is flat, and had Gaussian initial fluctuations with a scale-invariant spectrum. We discuss the constraints imposed on such theoretical models by the *COBE* measurements, by observations of galaxy clustering, and by the observed streaming motions of galaxies. When normalized to match the *COBE* results, models with $\Omega = 1$ and with more large-scale power than the standard cold dark matter (CDM) model predict lower streaming motions than are observed, but agree well with the dynamics of clustering on smaller scales. Unbiased $\Omega = 1$ CDM models fit the *COBE* data and the streaming motions, but are less easily reconciled with galaxy clustering data on either small or large scales. Spatially flat CDM models with $\Omega \sim 0.2$ and a cosmological constant require the mass to be substantially *more* clustered than the galaxies in order to be consistent with *COBE* and with observed streaming motions. They are then in conflict, however, with dynamical measurements on smaller scales.

Key words: cosmic microwave background – dark matter – large-scale structure of Universe.

1 INTRODUCTION

Ever since the microwave background radiation was first discovered, cosmologists have searched for temperature fluctuations in the hope of providing unambiguous evidence for the existence of inhomogeneities in the early Universe. It appears that these fluctuations have now been discovered by the *COBE* DMR experiment (Smoot *et al.* 1992, hereafter S92). The anisotropies have the frequency and spatial characteristics expected for primordial fluctuations in the background radiation and are consistent with the scale-invariant spectrum predicted by models of inflation (Bardeen, Steinhardt & Turner 1983, and references therein). Wright *et al.* (1992) compare the *COBE* results with Holtzman's (1989) grid of models and with previous upper limits on the anisotropies on a wide range of scales. They conclude that the *COBE* detection is consistent with previous anisotropy measurements and with a number of theoretical models of structure formation.

In this paper, we consider the implications of the *COBE* measurement for the confrontation between theory and observations of large-scale structure in the Universe. We specialize to the case of a spatially flat universe with scale-invariant adiabatic initial fluctuations as favoured by the *COBE* results (S92).* In Section 2 we relate the present-day amplitude of large-scale fluctuations in the mass distribution to the *COBE* measurements. Section 3 describes the relation between such large-scale fluctuations in the mass and clustering in the galaxy distribution. The constraints on theories of galaxy formation are discussed in Section 4.

2 ANISOTROPIES DETECTED BY *COBE*

On the angular scales probed by *COBE* ($\theta \gtrsim 7°$), temperature anisotropies in adiabatic theories arise from potential fluctuations in the early Universe (the Sachs–Wolfe effect, Sachs & Wolfe 1967). Specifying the background cosmology allows us to relate the temperature fluctuations to density fluctu-

*Spatial curvature becomes important on angular scales exceeding $\Omega_0/(1-\Omega_0)^{1/2}$ if $\Lambda = 0$ and beyond this scale there is no natural generalization of a scale-invariant spectrum. The observed scale-invariance of the temperature fluctuations thus argues for a spatially flat universe.

ations at the present epoch. Expanding the temperature fluctuations on the sky in spherical harmonics,

$$\frac{\Delta T}{T} = \sum_{\ell, m} a_\ell^m Y_\ell^m(\theta, \phi), \tag{1}$$

and assuming that the Universe is spatially flat, we can relate the angular power spectrum $C_\ell = \langle |a_\ell^m|^2 \rangle$ to the present-day power spectrum $P(k)$ of the mass fluctuations:

$$C_\ell = \frac{\Omega_0^{1.54}}{2\pi} \frac{H_0^4}{c^4} \int_0^\infty \frac{P(k)}{k^2} j_\ell^2(kr_0)\, dk,$$

$$r_0 = \frac{c}{H_0 \Omega_0^{1/2}} \int_1^\infty (x^3 + \Omega_0^{-1} - 1)^{-1/2}\, dx \tag{2}$$

(Peebles 1984). In this equation, Ω_0 is the mean matter density at the present epoch divided by the critical density and H_0 is Hubble's constant. In a spatially flat universe, these parameters are related to the cosmological constant Λ by $\Lambda = 3(1-\Omega_0)H_0^2$. When $\Lambda > 0$, the lowest order multipoles differ slightly from equation (2) because of the deviation of the fluctuation growth rate from its Einstein–de Sitter form (see Kofman & Starobinsky 1985 and Gorski, Silk & Vittorio 1992 for further details); we ignore this effect in this paper since it is smaller than the observational uncertainties.

For mass fluctuations with a scale-invariant spectrum,

$$P(k) = Bk, \tag{3}$$

equation (2) gives

$$C_\ell = \frac{\Omega_0^{1.54}}{4\pi} \frac{H_0^4}{c^4} \frac{B}{\ell(\ell+1)}. \tag{4}$$

The *COBE* team have fitted equation (4) to their data (excluding the dipole and quadrupole coefficients) and find a best-fitting amplitude

$$Q_{\rm rms} = \left(\frac{5C_2}{4\pi}\right)^{1/2} T_0 = 16.7 \pm 4.6\ \mu\text{K} \tag{5}$$

(S92), where T_0 is the mean temperature of the background radiation ($T_0 = 2.735 \pm 0.006$ K, Mather *et al.* 1990) and the error includes the contribution from 'cosmic variance', that is, the fact that the power in the observed sky will not equal the ensemble mean power for all possible skies in our Universe. For the rms measured quadrupole S92 give $13 \pm 4\ \mu$K, leading to $Q_{\rm rms} = 13 \pm 6.0\ \mu$K when cosmic variance is included. A weighted average of these two is then $Q_{\rm rms} = 15.3 \pm 3.7\ \mu$K which we take as the *COBE* measurement of the large-scale fluctuation amplitude. For a scale-invariant spectrum, this value of $Q_{\rm rms}$ gives a variance of 31 μK for a 10° FWHM Gaussian beam, in excellent agreement with the value of $30 \pm 5\ \mu$K measured by *COBE*. We conservatively use the larger error on $Q_{\rm rms}$ in comparing with theoretical models. (Normalizing to the 10° fluctuations, including their cosmic variance, leads to essentially identical conclusions to those given below.)

From equation (4), we predict

$$Q_{\rm rms}/T_0 = \left(\frac{5}{6\pi^2}\right)^{1/2} \left(\frac{H_0}{2c}\right)^2 \Omega_0^{0.77} B^{1/2}, \tag{6}$$

allowing the *COBE* measurement to be related to B. *COBE* also provides direct evidence that equation (3) applies on wavenumbers $k \lesssim \ell_{\rm max}/r_0$ where $\ell_{\rm max} \sim 20$ is fixed by the resolution of the *COBE* DMR and $r_0 \approx 2c\Omega_0^{0.41}/H_0$ (i.e. for wavenumbers less than $0.003\Omega_0^{0.41} h\,\text{Mpc}^{-1}$†).

Having fixed the large-scale fluctuation amplitude from equations (5) and (6), we can now discuss how the *COBE* results relate to other observations of large-scale structure in the Universe and so to theoretical models such as the CDM model. In any given theory, the fluctuation amplitude given by *COBE* determines that on all other scales and the implied inhomogeneities must be consistent with dynamical and other constraints if the model is to be viable.

3 POWER SPECTRA AND FLUCTUATIONS IN THE GALAXY DISTRIBUTION

Structure in the galaxy distribution has been detected on scales up to $\sim 50\ h^{-1}$ Mpc, or wavenumbers $k \lesssim 0.02\ h\,\text{Mpc}^{-1}$ (Maddox *et al.* 1990a; Geller & Huchra 1990; Efstathiou *et al.* 1990a; Loveday *et al.* 1992). The structure seen by *COBE* is more than an order of magnitude larger. Thus the *COBE* results can only be compared with other data via a model for the scale dependence of fluctuations.

We have adopted the following parametric form to extrapolate the power spectrum of equation (3) to smaller scales:

$$P(k) = \frac{Bk}{\{1 + [ak + (bk)^{3/2} + (ck)^2]^\nu\}^{2/\nu}}, \tag{7}$$

where $a = (6.4/\Gamma)\ h^{-1}$ Mpc, $b = (3.0/\Gamma)\ h^{-1}$ Mpc, $c = (1.7/\Gamma)\ h^{-1}$ Mpc and $\nu = 1.13$. This functional form is, of course, motivated by the CDM model. For 'standard' CDM with $\Omega_0 = 1$, equation (7) provides an accurate fit to the linear power spectrum if Γ is set equal to h and the baryon density is small (Bond & Efstathiou 1984). In fact, a wide class of theoretical models can be fitted by equation (7). Low-density CDM models in a spatially flat universe (i.e. $\Lambda > 0$) are accurately described by equation (7) with $\Gamma = \Omega_0 h$. CDM models with decaying neutrinos (e.g. Bond & Efstathiou 1991) are well described by (7) if

$$\Gamma \approx \Omega_0 h / [0.861 + 3.8(m_{10}\tau_{\rm d})^{2/3}]^{1/2},$$

where m_{10} is the neutrino mass in units of 10 keV and $\tau_{\rm d}$ is its lifetime in years. Even scale-invariant models with a mixture of hot and cold dark matter can be approximately described by equation (7) provided Ω_ν is not too large. For example, the power spectra of $\Omega_0 = 1$, $h = 0.5$, CDM-dominated universes in which one species of massive neutrino contributes Ω_ν in the range 0.13–0.37 (van Dalen & Schaeffer 1992), and hence with m_ν ranging from 3–9 eV, can roughly be fitted with $\Gamma \sim 0.2(\Omega_\nu/0.3)^{-1/2}$ over the k-band probed by large-scale structure observations.

Thus, by adjusting the parameter Γ, we can vary the amount of large-scale power relative to standard CDM. Furthermore, if $\Gamma \approx 0.2$, equation (7) provides a good fit to the shape of the large-scale correlations observed in surveys of galaxies and clusters (Efstathiou, Sutherland & Maddox 1990b; Efstathiou *et al.* 1992). Fig. 1 compares angular

†Here and below we write $H_0 = 100h\ \text{km s}^{-1}\ \text{Mpc}^{-1}$.

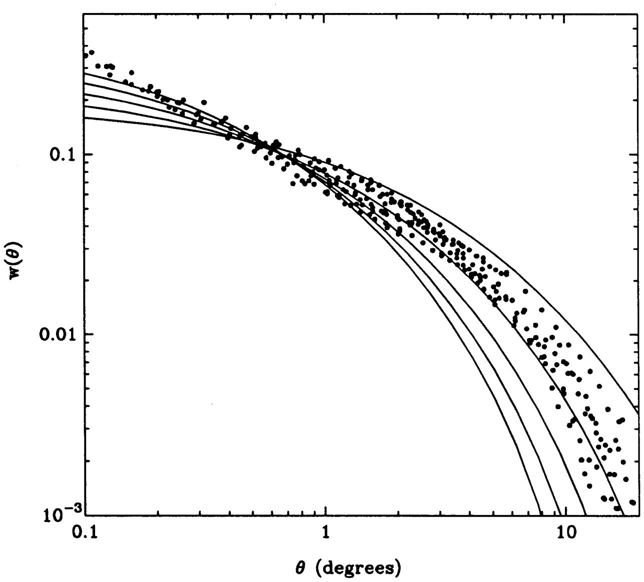

Figure 1. Angular correlation function from various magnitude slices from the APM Galaxy Survey (Maddox *et al.* 1990a) scaled to a depth at which 1° corresponds to a physical scale of $\sim 5\ h^{-1}$ Mpc. The curves show the linear theory predictions from the power spectrum of equation (7) (normalized to $\sigma_8 = 1$) for models with $\Gamma = 0.5$ (lowest curve at $\theta \gtrsim 1°$) 0.4, 0.3, 0.2, 0.1 (highest curve at $\theta \gtrsim 1°$). These data indicate $\Gamma \sim 0.2$, but corrections for systematic errors in the APM catalogue could allow values as large as $\Gamma \sim 0.3$ (Maddox, Efstathiou & Sutherland 1992, in preparation). The parameter Γ is related to the measure of 'excess power' (E) introduced by Wright *et al.* (1992) by $E = 3.4\sigma_{25}/\sigma_8 = 0.95(\Gamma/0.5)^{-0.3}$, where σ_{25} is the rms density fluctuation in spheres of radius 25 h^{-1} Mpc.

correlations computed from equation (1) with measurements from the APM Galaxy Survey (Maddox *et al.* 1990a). Taking into account possible systematic errors in the APM data (which could lead to overestimation of $w(\theta)$ at large θ; Maddox, Efstathiou & Sutherland 1990b), we conclude that large-scale structure in the galaxy distribution can be described by equation (7) if Γ lies in the range $0.15 \lesssim \Gamma \lesssim 0.3$. Equation (7) is thus a convenient parametrization of the fluctuation spectrum which is scale-invariant on large scales, is a good approximation to many models of interest, and can match the shape of the observed galaxy correlation function.

With the power spectrum normalization determined from *COBE* by equation (6), we can use equation (7) to compute the variance, σ_8^2, of the mass overdensity within spheres of radius 8 h^{-1} Mpc as a function of our parameters Γ and Ω_0. Fluctuations in the distribution of optically selected galaxies give $(\sigma_8^2)_{\rm gal} \approx 1$ (Davis & Peebles 1983), thus our estimates of σ_8 are measures of a 'biasing factor' $b_8 = 1/\sigma_8$. Fig. 2(a) shows the *COBE* constraints on σ_8 as a function of Γ for models with $\Omega_0 = 1$. The standard CDM model requires $b_8 \approx 0.93$ with a 1σ range of 0.75–1.22. This is considerably lower than the values, $b_8 \sim 1.5$–2.5, that have been assumed in most discussions of the CDM model (e.g. see Davis *et al.* 1985; Bardeen *et al.* 1986; Frenk *et al.* 1990), although $b_8 = 1.8$ is only just excluded at the 2σ level given the uncertainty on the *COBE* result. However, Couchman & Carlberg (1992) have recently simulated an $\Omega = 1$, $b_8 = 0.8$, CDM model with results that appear compatible with observation on a wide range of scales. This point is discussed further in the next section.

For $\Omega_0 = 1$, models with Γ in the range suggested by the shape of the APM galaxy correlations are consistent with the

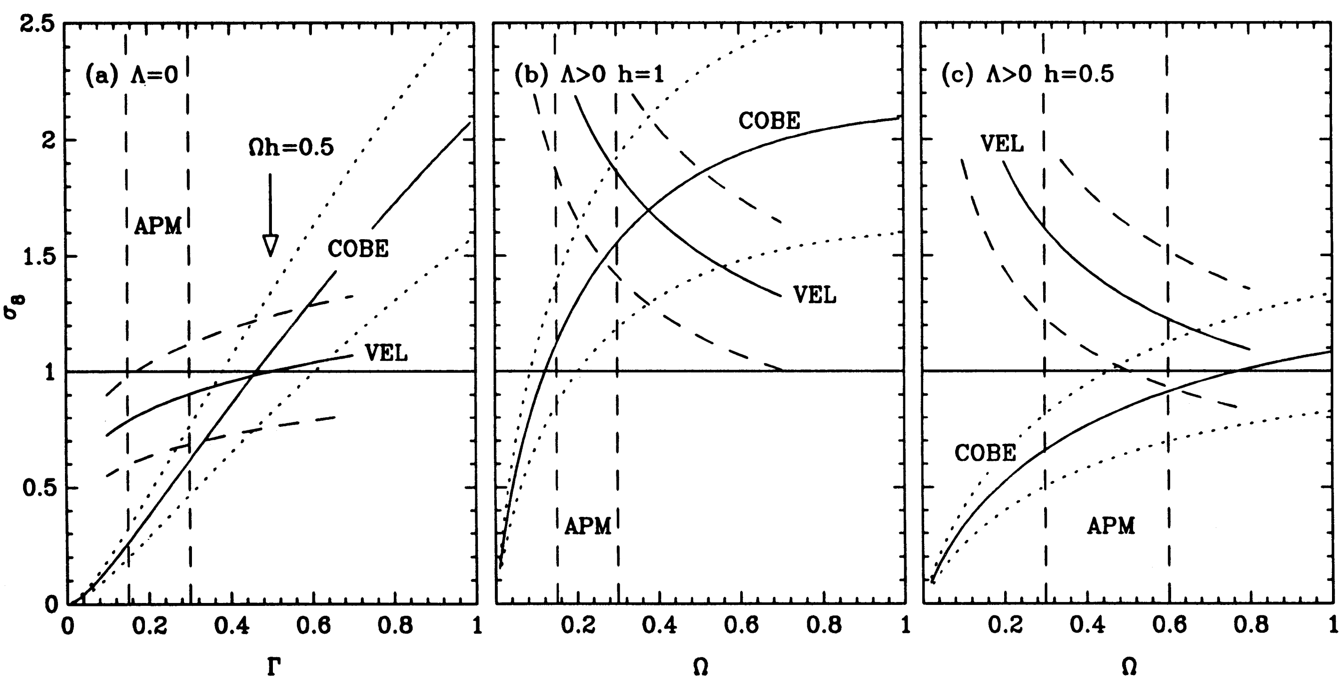

Figure 2. Constraints on $\sigma_8 = 1/b_8$ imposed by the *COBE* detection of anisotropies in the microwave background radiation. (a) shows results for $\Omega_0 = 1$. The solid curve labelled COBE gives the value of σ_8 that reproduces the *COBE* amplitude as a function of the large-scale structure parameter Γ. The dotted lines show the range allowed by one standard deviation on the *COBE* measurement. The vertical lines marked APM delimit the range of Γ for which the shape of the mass correlation function is consistent with that of the galaxy correlation function in the APM survey. The curves labelled VEL show the amplitude required to match large-scale peculiar motions (equation 9), again with a 1σ range. The position of standard CDM ($\Omega_0 = 1$, $h = 0.5$) is marked by the arrow. (b) and (c) show similar curves for low-density, spatially flat, CDM models with $h = 1$ and 0.5 respectively.

COBE results if $b_8 \sim 2$, though the range of allowed values is wide, $1.4 \lesssim b_8 \lesssim 4$, when the uncertainties in both Γ and the *COBE* normalization are taken into account. [$\Omega_0 = 1$ hot/cold hybrid models need slightly smaller biasing factors, $b_8 \approx (1.5 \pm 0.3)(\Omega_\nu/0.3)^{-0.15}$ for Ω_ν in the range 0.13–0.37, to agree with the *COBE* and APM results.] Figs 2(b) and (c) show the *COBE* constraints on b_8 for spatially flat models with $\Omega_0 < 1$, and with $h = 1$ and 0.5 respectively. In this model the constraints on b_8 depend on both Ω_0 and Γ. We have set $\Gamma = \Omega_0 h$, as appropriate for low-density CDM models with $\Omega_b \ll \Omega_0$, and plotted b_8 as a function of Ω_0. For the range of Γ allowed by observations of large-scale structure, we require $b_8 \sim 0.5$–1.1 for $h = 1$, and $b_8 \sim 0.9$–2.0 for $h = 0.5$. Thus solutions where galaxies are unbiased with respect to the mass are clearly acceptable.

4 DISCUSSION

Three classes of scale-invariant model can be discussed in the framework we have introduced. Normalized to the *COBE* observations, they are:

(1) $\Omega_0 = 1$, high-b_8 models with extra large-scale power relative to 'standard' CDM ($b_8 \sim 2, \Gamma \sim 0.2, \Lambda = 0$);

(2) $\Omega_0 = 1$, low-b_8, 'standard' CDM ($b_8 \sim 1$, $\Gamma = h \sim 0.5$, $\Lambda = 0$), and

(3) low-density, low-b_8 CDM models ($b_8 \sim 1$, $\Omega_0 h \sim 0.2$, $\Lambda \neq 0$).

A precise measurement of b_8 could therefore distinguish between some of these models. For example, if $b_8 \sim 2$, then the *COBE* results are exactly what is expected if the observed galaxy correlation function is proportional to that of the underlying mass (Bardeen, Bond & Efstathiou 1987; Efstathiou 1991); with this value of b_8 the 'standard' CDM power spectrum is excluded by *COBE* at the 2σ level. However, if $b_8 \sim 1$, we are forced either to invoke non-zero Λ, or to accept models in which non-linear biasing masks the dynamical effects of large mass fluctuations on small scales (Couchman & Carlberg 1992) *and* alters the shape of the galaxy correlation function on large scales (Babul & White 1991; Efstathiou 1992; Bower *et al.* 1992). Unfortunately, the value of b_8 is poorly known because of uncertainties in the data on moderately large scales where the dynamics are simple, and because of uncertainties in possible non-linear effects on small scales where the data are good. We now summarize some of the techniques and arguments that have been used to estimate its value.

(A) *Streaming motions.* Galaxy peculiar velocities measure mass fluctuations directly and so can provide a direct measure of σ_8 [and, in principle, of the shape of the power spectrum $P(k)$]. Various versions of the technique have been applied by a number of authors (see e.g. Kaiser & Lahav 1989; Gorski *et al.* 1989; Bertschinger *et al.* 1990). Bertschinger *et al.* give estimates for the average velocities of the mass within spheres of radius 40 and 60 h^{-1} Mpc centred on the Local Group. Comparing these numbers, 388 ± 67 and 327 ± 82 km s^{-1} respectively, with the rms predictions for such spheres from the power spectrum of equation (7), we find that they require

$$\sigma_8 \sim 1.15 \Omega_0^{-0.6} \left(\frac{\Gamma}{0.5}\right)^{0.7}. \tag{8}$$

The $\pm 1\sigma$ range in the coefficient here is approximately 0.8–2.2 and is large because of the 'cosmic variance' expected in the measurement of a single bulk streaming velocity. Given this uncertainty, the constraints on σ_8 from equation (8) have a large overlap with those implied by the *COBE* observations for all three of the models discussed at the beginning of this section.

While the bulk motion of a sphere is dominated by long-wavelength fluctations, the differential motions within it are due to smaller structures. Kaiser *et al.* (1991) have used linear theory to predict peculiar velocities from smoothed maps of the distribution of *IRAS* galaxies. Comparison with observed peculiar velocities led them to $b_{IRAS}/\Omega_0^{0.6} = 1.16 \pm 0.21$, where b_{IRAS} is the ratio of smoothed *IRAS* overdensity to smoothed mass overdensity. Saunders, Rowan-Robinson & Lawrence (1992) find that for this same volume of space, but for a larger *IRAS* galaxy sample, the rms galaxy overdensity in cubes of side 30 h^{-1} Mpc (roughly the smoothing scale adopted by Kaiser *et al.*) is 0.43 ± 0.08, implying an rms mass overdensity in these same cubes of $0.37 \pm 0.09 \Omega_0^{-0.6}$ assuming that b_{IRAS} is independent of scale. Using the power spectrum of equation (7) to link this to the fluctuations within 8 h^{-1} Mpc spheres we obtain

$$\sigma_8 = 1.0 \pm 0.24 \Omega_0^{-0.6} \left(\frac{\Gamma}{0.5}\right)^{0.20}. \tag{9}$$

The formal error on this result is considerably smaller than in equation (8) and consequently the constraints implied are more interesting. Model (1) (with $\Omega_0 = 1$, $b_8 = 2$ and $\Gamma = 0.2$) is excluded at more than the 2σ level because it predicts peculiar velocities which are too low. However, a small increase in Γ allows an increase in σ_8 and so reduces the disagreement to an acceptable level. The *COBE*, APM, and streaming constraints are all satisfied for $\Gamma \approx 0.28$ and $b_8 \approx 1.5$. Model (2) agrees well with the streaming constraint for the parameters given above, but the shape of its mass correlation function disagrees, of course, with that of the galaxies in the APM survey. For model (3) with $\Omega_0 h = 0.2$, equation (9) requires $b_8 \sim 0.5$ for $h = 1$ and $b_8 \sim 0.7$ for $h = 0.5$. Comparison with Figs 2(b) and (c) shows that these disagree with the *COBE* data at nearly the 2σ level. Again the problem is that the *COBE* normalization predicts mass fluctuations which are too small on the relevant scale (~ 30 h^{-1} Mpc), and again it can be reduced by increasing Γ to the limit allowed by the APM correlations. It is unclear how sensitive these results are to the specific model for biasing adopted by Kaiser *et al.* Nevertheless, this type of comparison between velocity and density fields seems a promising way of discriminating between theoretical models.

(B) *Small-scale dynamics.* The dynamical properties of groups and clusters of galaxies suggest that the mass per galaxy within the virialized regions ($r \sim 1$ h^{-1} Mpc) is only about 20 per cent of that needed globally in a flat universe (see e.g. Faber & Gallagher 1979, for a review). This would imply that the galaxy distribution is strongly biased towards such dense regions (relative to the mass) if indeed $\Omega_0 = 1$. This was the original motivation for the idea of biased galaxy formation, and N-body simulations with a particular prescription for biasing led to the conclusion that $b_8 \sim 2$ was necessary to explain the observed rms peculiar velocities of galaxies and the low mass-to-light ratios of clusters of galaxies in a flat universe (Davis *et al.* 1985; Frenk *et al.* 1990). This clearly favours model (1) over model (2), and suggests that model (3) requires $b_8 \approx 1$ for $h=1$ and $b_8 > 1$ for $h=0.5$, in conflict with the lower b_8 values derived above from streaming motions. However, a recent simulation by Couchman & Carlberg (1992) suggests that the positions and velocities of galaxies can both be biased relative to the dark matter in such a way that small-scale random motions and cluster mass-to-light ratios are compatible with observation even for b_8 as small as 0.8. The biases required are large, and it is difficult to assess the reliability of the results since the analysis of Couchman & Carlberg is based on a plausible but highly schematic recipe for identifying galaxies within a single simulation containing a single quasi-equilibrium cluster. Nevertheless, the possibility that models with $\Omega=1$ and $b_8 \sim 1$ can be compatible with observation must clearly be taken seriously until we understand galaxy formation and the non-linear evolution of clustering much more fully than is currently the case.

(C) *Abundances of clusters.* If $\Omega_0=1$ and $b_8 \sim 1$ then clumps with masses and sizes similar to rich clusters of galaxies should be abundant in the present Universe. This conclusion is almost independent of the shape of the fluctuation spectrum because the mass contained within an 8 h^{-1} Mpc sphere ($6 \times 10^{14}\ h^{-1}\ M_\odot$) is equal to the mass contained within the Abell radius of a rich cluster. For example, according to Hughes (1989) the Coma cluster contains about $7 \times 10^{14}\ h^{-1}\ M_\odot$ within this radius. Thus if $\sigma_8 \sim 1$, a substantial fraction of the mass of the Universe should have collapsed into objects with the mass of the Coma cluster, whereas in fact there appears to be only about one such object per (100 h^{-1} Mpc)^3. Versions of this argument have been used by Bardeen *et al.* (1986) to infer $b_8 \sim 1.7$ and by Frenk *et al.* (1990) to infer $b_8 \sim 2$–2.5. These numbers agree well with model (1) and disagree with model (2). If $\Omega_0=0.2$, the mass within the normalization sphere is five times smaller, and similar arguments lead to estimates of $b_8 \approx 1$. This agrees with model (3) as defined at the beginning of this section, and with the standard idea that galaxies should trace the mass in an open universe. However, it is again inconsistent with the normalization required for model (3) by equation (9); large enough streaming motions can only be obtained in this model at the expense of overproducing rich clusters. These conclusions may depend sensitively on how rich clusters are defined observationally and on how cluster abundances are calculated theoretically. For example, Bond & Myers (1992) conclude that CDM models with $b_8 \sim 1.1$–1.5 are favoured by X-ray observations of rich clusters of galaxies.

For both model (2) and model (3) large-scale motions imply significantly smaller values of b_8 than those usually inferred from small-scale dynamics or cluster abundances. This discrepancy is independent of the *COBE* results and its resolution seems to require jettisoning either the peculiar velocity data or the theoretical arguments of (B) and (C). One of the most interesting conclusions to emerge from the *COBE* measurements is that $\Omega=1$ CDM variants with extra large-scale power ($\Gamma \lesssim 0.3$) require $b_8 \gtrsim 1.4$. For values near this limit our model (1) can be consistent with observed streaming motions and with conventional interpretations of smaller scale dynamics and of cluster abundances. However, rather than the 'clustering' tests described above, the best way to discriminate between models may well prove to be through mapping of the microwave fluctuations at smaller angular scales where the temperature power spectrum becomes dependent on the matter content of the Universe [see Bond & Efstathiou (1987) for detailed theoretical predictions].

Finally, even if we demonstrate convincingly that $b_8 \sim 1.7$, this would not rule out standard CDM at a high significance level ($\lesssim$two standard deviations) given the current uncertainties in the *COBE* measurements. At this level of precision we also have to consider carefully a number of small effects (each at the ~5–20 per cent level) that might combine to bring the model into better agreement with the data. Amongst such effects we mention: (i) uncertainties in the Hubble constant, which could perhaps be as low as $H_0=40$ km s^{-1} Mpc (e.g. Narayan 1991, and references therein); (ii) most models of inflation predict small departures from an exact, $P(k) \propto k$, spectrum in the sense that σ_8 is reduced by 10–20 per cent for fixed quadrupole amplitude (Salopek, Bond & Bardeen 1989); (iii) up to a few per cent of the amplitude measured by *COBE* could be due to gravitational waves generated during inflation, rather than to the associated scalar field fluctuations (Starobinsky 1985); (iv) variations in the baryon abundance within the range allowed by primordial nucleosynthesis can lead to small changes in the shape of the CDM power spectrum, and (v) small residual contamination from galactic dust or synchrotron emission may have inflated the *COBE* measurements.

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REFERENCES

Babul, A. & White, S. D. M., 1991. *Mon. Not. R. astr. Soc.*, **253,** 31P.
Bardeen, J. M., Steinhardt, P. J. & Turner, M. S., 1983. *Phys. Rev. D*, **28,** 679.
Barden, J. M., Bond, J. R. & Efstathiou, G., 1987. *Astrophys. J.*, **321,** 18.
Bardeen, J. M., Bond, J. R., Kaiser, N. & Szalay, A. S., 1986. *Astrophys. J.*, **304,** 15.
Bertschinger, E., Dekel, A., Faber, S. M., Dressler, A. & Burstein, D., 1990. *Astrophys. J.*, **364,** 370.
Bond, J. R. & Efstathiou, G., 1984. *Astrophys. J. Lett.*, **285,** L45.
Bond, J. R. & Efstathiou, G., 1987. *Mon. Not. R. astr. Soc.*, **226,** 655.
Bond, J. R. & Efstathiou, G., 1991. *Phys. Lett. B*, **265,** 245.
Bond, J. R. & Myers, S. T., 1992. In: *Trends in Astroparticle Physics*, p. 262, eds Cline, D. & Peccei, R., World Scientific, Singapore.
Bower, R. G., Coles, P., Frenk, C. S. & White, S. D. M., 1992. *Mon. Not. R. astr. Soc.*, , in press.
Couchman, H. M. P. & Carlberg, R. G., 1992. *Astrophys. J.*, **389,** 453.
Davis, M. & Peebles, P. J. E., 1983. *Astrophys. J.*, **267,** 465.
Davis, M., Efstathiou, G., Frenk, C. S. & White, S. D. M., 1985. *Astrophys. J.*, **292,** 371.
Efstathiou, G., 1991. In: *Observational Tests of Inflation*, p. 425, eds Shanks, T., Banday, A. J., Ellis, R. S., Frenk, C. S. & Wolfendale, A. W., NATO ASI Series, Kluwer, Dordrecht.

Efstathiou, G., 1992. *Mon. Not. R. astr. Soc.*, **256**, 43P.
Efstathiou, G., Dalton, G. D., Maddox, S. J. & Sutherland, W. J., 1992. *Mon. Not. R. astr. Soc.*, in press.
Efstathiou, G., Kaiser, N., Saunders, W., Rowan-Robinson, M., Lawrence, A., Ellis, R. S. & Frenk, C. S., 1990a. *Mon. Not. R. astr. Soc.*, **247**, 10P.
Efstathiou, G., Sutherland, W. J. & Maddox, S. J., 1990b. *Nature*, **348**, 705.
Faber, S. M. & Gallagher, J. S., 1979. *Ann. Rev. Astr. Astrophys.*, **17**, 135.
Frenk, C. S., White, S. D. M., Efstathiou, G. & Davis, M., 1990. *Astrophys. J.*, **351**, 10.
Geller, M. J. & Huchra, J. P., 1990. *Science*, **246**, 897.
Gorski, K., Silk, J. & Vittorio, N., 1992. *Phys. Rev. Lett.*, **50**, 620.
Gorski, K., Davis, M., Strauss, M. A., White, S. D. M. & Yahil, A., 1989. *Astrophys. J.*, **344**, 1.
Holtzman, J. A., 1989. *Astrophys. J. Suppl.*, **71**, 1.
Hughes, J. P., 1989. *Astrophys. J.*, **337**, 21.
Kaiser, N. & Lahav, O., 1989. *Mon. Not. R. astr. Soc.*, **237**, 129.
Kaiser, N., Efstathiou, G., Ellis, R. S., Frenk, C. S., Lawrence, A., Rowan-Robinson, M. & Saunders, W., 1991. *Mon. Not. R. astr. Soc.*, **252**, 1.
Kofman, L. A. & Starobinsky, A. A., 1985. *Soviet Astr. Lett.*, **11**, 271.
Loveday, J., Efstathiou, G., Peterson, B. A. & Maddox, S. J., 1992. *Astrophys. J.*, in press.
Maddox, S. J., Efstathiou, G., Sutherland, W. J. & Loveday, J., 1990a. *Mon. Not. R. astr. Soc.*, **242**, 43P.
Maddox, S. J., Efstathiou, G. & Sutherland, W. J., 1990b. *Mon. Not. R. astr. Soc.*, **246**, 433.
Mather, J. C. *et al.*, 1990. *Astrophys. J. Lett.*, **354**, L37.
Narayan, R., 1991. *Astrophys. J. Lett.*, **378**, L5.
Peebles, P. J. E., 1984. *Astrophys. J.*, **284**, 439.
Sachs, R. K. & Wolfe, A. M., 1967. *Astrophys. J.*, **147**, 73.
Salopek, D. S., Bond, J. R. & Bardeen, J. M., 1989. *Phys. Rev. D*, **40**, 1753.
Saunders, W., Rowan-Robinson, M. & Lawrence, A., 1992. *Mon. Not. R. astr. Soc.*, **258**, 134.
Smoot, G. F. *et al.*, 1992. *Astrophys. J. Lett.*, in press (S92).
Starobinsky, A. A., 1985. *Soviet Astr. Lett.*, **11**, 133.
van Dalen, A. & Schaeffer, R. K., 1992. Preprint.
Wright, E. L. *et al.*, 1992. *Astrophys. J.*, in press.